## Examination Quantum Chemistry. 09/03/2020, 14.30-18.00

- This exam has 7 questions on 4 pages;
- Explain all your answers (no explanation, no points!).
- Do not forget you name and s-number!
- Review: Dr. J.E.M.N. Klein
- Grade: $\frac{\# p+7}{7}$, maximum number of points is 63 .

Question 1 (8 points (2,2,2,2))
The overlap between two hydrogen s-orbitals, p-orbitals and an s- and p-orbital is calculated as a function of the $\mathrm{H}-\mathrm{H}$ distance. Unfortunately, the labels got lost.

R

S-S

$p-p$

a) Explain which curve belongs to which overlap.
b) Sketch the energy difference between the bonding and the anti-bonding $\sigma$ orbitals for $\mathrm{H}_{2}$ as a function of the interatomic distance. Explain why you have drawn the curve as you did.
c) Is the restricted Hartree-Fock method appropriate to describe this dissociation?
d) Explain what you expect for the calculated dissociation energy using the restricted Hartree-Fock method compared to experiment. Do you expect it to be too small, too high, or correct, and why?

Question 2 (8 points (2,2,2,2) )
a) Mention one difference between HF and DFT.
b) Mention one functional that can be used in a DFT calculation.
c) Does the following two-electron wavefunction, $\Psi(1,2)=|a \bar{a}|+|b \bar{b}|$, obey the Pauli principle?
d) Is the wavefunction mentioned in question c) normalised to 1 ? If not, normalise it to 1 (the orbitals $a$ and $b$ are orthonormal).

Question 3 (8 points (1,1,2,2,2))
a) A system is in a particular quantum state described by the wavefunction $\psi=\frac{1}{\sqrt{2}}(f+g)$. The functions $f$ and $g$ are eigenfunctions of the operator A: $A f=a_{1} f ; A g=a_{2} g \quad\left(a_{1} \neq a_{2}\right)$.
i. Is the function $\psi$ an eigenfunction of operator A? If so, what is the eigenvalue?
ii. Calculate the expectation value for operator A .
b) Explain why the dissociation energy for $D_{2}\left(D={ }^{2} H\right)$ is slightly different from that of $\mathrm{H}_{2}$.
c) Give a short description of the CI method.
d) Mention an example of a chemical problem for which the use of the MCSCF method is recommended.

Question 4 (10 points (2,2,2,2,2) )
Hückel theory is used for the study of the following molecule:
a) Give the secular determinant for this system.

The results of a Hückel calculation are summarised in the Table.


Table

|  | MO 1 | MO 2 | MO 3 | MO 4 | MO 5 | MO 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\varepsilon$ | $\alpha+2.11 \beta$ | $\alpha+1.00 \beta$ | $\alpha+0.62 \beta$ | $\alpha-0.25 \beta$ | $\alpha-1.62 \beta$ | $\alpha-1.86 \beta$ |
| $\mathrm{C}_{1}$ | -0.2473 | -0.5000 | 0.0000 | -0.7495 | 0.0000 | -0.3566 |
| $\mathrm{C}_{2}$ | -0.5230 | -0.5000 | 0.0000 | 0.1904 | 0.0000 | 0.6635 |
| $\mathrm{C}_{3}$ | -0.4294 | 0.0000 | -0.6015 | 0.3505 | 0.3717 | -0.4390 |
| $\mathrm{C}_{4}$ | -0.3851 | 0.5000 | -0.3717 | -0.2795 | -0.6015 | 0.1535 |
| $\mathrm{C}_{5}$ | -0.3851 | 0.5000 | 0.3717 | -0.2795 | 0.6015 | 0.1535 |
| $\mathrm{C}_{6}$ | -0.4294 | 0.0000 | 0.6015 | 0.3505 | -0.3717 | -0.4390 |

b) What is the total Hückel energy for this system?
c) The energy of the double bond in ethene is $\alpha+\beta$. What is the resonance or delocalisation energy for this molecule?
d) Calculate the charge of the $\mathrm{C}_{1}$ atom.
e) Calculate the $\mathrm{C}_{2}-\mathrm{C}_{3} \pi$-bond order.

Question 5 (11 points (2,2,2,2,2,1))
The bonding orbital of $\mathrm{H}_{2}{ }^{+}$is written as, $\psi=c_{1} s_{A}+c_{2} p_{A}+c_{3} s_{B}+c_{4} p_{B}$, with $\mathrm{s}_{\mathrm{A} / \mathrm{B}}$ the 1 s orbital centred on atom $\mathrm{A} / \mathrm{B}$, and $\mathrm{p}_{\mathrm{A} / \mathrm{B}}$ the p-orbital, aligned along the intermolecular axis, on atom A/B, respectively. The atomic orbitals are considered to be orthonormal and the following integrals have been calculated (all in Hartree):
$\int s_{A} H s_{A} d \tau=\int s_{B} H s_{B} d \tau=-0.5$
$\int p_{A} H p_{A} d \tau=\int p_{B} H p_{B} d \tau=-0.125$
$\int s_{A} H p_{A} d \tau=\int s_{B} H p_{B} d \tau=0.0$
$\int s_{A} H s_{B} d \tau=-0.4$
$\int s_{A} H p_{B} d \tau=\int p_{A} H s_{B} d \tau=-0.3$
$\int p_{A} H p_{B} d \tau=-0.2$
a) Give the secular determinant for this problem.

The use of symmetry can greatly reduce the complexity of the problem. We treat the molecule in the subgroup $\mathrm{C}_{\mathrm{i}}$ of the full point group. The character table for $\mathrm{C}_{\mathrm{i}}$ is:

| $C_{\mathrm{i}}=S_{2}$ <br> $(\overline{1})$ | $E$ | $i$ |  |  |
| :--- | :---: | :---: | :---: | :---: |
| $\mathrm{~A}_{\mathrm{g}}$ | 1 | 1 | $R_{x}, R_{y}, R_{z}$ | $x^{2}, y^{2}, z^{2}$, <br> $x y, x z, y z$ |
| $\mathrm{~A}_{\mathrm{u}}$ | 1 | -1 | $x, y, z$ |  |

b) Make the symmetry adapted linear combinations (SALCs) of the basis functions that we have in this case, and indicate according to which irreducible representation they transform.
c) Give the secular determinant in this SALC basis.
d) Determine the ground state energy of $\mathrm{H}_{2}{ }^{+}$.
e) Calculate the ground state wavefunction for $\mathrm{H}_{2}{ }^{+}$. Express the final orbital in terms of the original atomic orbital basis.
f) What would happen with the energy if we add a set of d-orbitals to each hydrogen atom?

Question 6 (8 points (2,2,2,2) )
One spinorbital of the hydrogen atom can be written as:

$$
\psi(r, \vartheta, \phi)=\alpha N r^{3} e^{-r / 4} \sin ^{3} \vartheta e^{-3 i \phi} .
$$

a) What do the variables $r, \vartheta, \phi$ of the function $\psi(r, \vartheta, \phi)$ indicate?
b) How many radial and angular nodes has this orbital?
c) What are the four quantum numbers for an electron described by this wavefunction?
d) Is there also a hydrogen orbital that is degenerate with this orbital, that has 4 angular nodes?

Question 7 (10 points (2,2,2,2,2))
Given the character table of the $\mathrm{C}_{3 \mathrm{v}}$ point group.

| $C_{3 \mathrm{v}}$ <br> $(3 m)$ | $E$ | $2 C_{3}$ | $3 \sigma_{\mathrm{v}}$ |  |  |
| :--- | ---: | ---: | ---: | :--- | :--- |
| $\mathrm{A}_{1}$ | 1 | 1 | 1 | $z$ | $x^{2}+y^{2}, z^{2}$ |
| $\mathrm{~A}_{2}$ | 1 | 1 | -1 | $R_{z}$ |  |
| E | 2 | -1 | 0 | $(x, y)\left(R_{x} R_{y}\right)$ | $\left(x^{2}-y^{2}, 2 x y\right)(x z, y z)$ |

a) How many classes does the $\mathrm{C}_{3 \mathrm{v}}$ point group have?
b) Show that $\mathrm{C}_{3}{ }^{+}$and $\mathrm{C}_{3}{ }^{-}$are conjugated elements.
c) Is the $\mathrm{C}_{3 \mathrm{v}}$ point group an Abelian group?

A conformation of the $\mathrm{H}_{3} \mathrm{Si}-\mathrm{CH}_{3}$ molecule belongs to the $\mathrm{C}_{3 \mathrm{v}}$ point group (with the $\mathrm{C}_{3}$ axis aligned along the z -axis).

## $\mathrm{H}_{3} \mathrm{Si}-\mathrm{CH}_{3}$

d) Argue on the basis of symmetry whether this molecule can have a permanent dipole.
e) According to which irreducible representation does the following orbital transform?


The End

